HIGH QUALITY ISOSURFACE CONSTRUCTION FROM VOLUMETRIC DATA SAMPLED WITH A FACE-CENTERED CUBIC LATTICE

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ABSTRACT

In this paper, we propose a method for constructing an isosurface from volumetric data sampled with a face-centered cubic lattice. In general, the isosurface is constructed as a polyhedron composed of connected triangle patches. Most existing methods generate each vertex of the triangle patches on the edge of a polyhedral cell. The proposed method generates the vertices inside of the polyhedral cells. Our method enables construction of a high quality isosurface because it generates many good aspect ratio triangle patches of an almost uniform size. We experimentally compared the resulting surface of our method with those of existing methods, and have thus demonstrated the effectiveness of our method

1. INTRODUCTION

Volumetric data is widely used in many disciplines such as biomedical science, computer graphics, and visualization. Computed tomography is a typical use of volumetric data. The value of each lattice point is the degree of x-ray beam attenuation at that point. These points are samples with an orthogonal cubic lattice. Visualization of such volumetric data is important to understand their geometrical properties. There are many ways of visualizing three-dimensional volumetric data. Isosurface representation is the most common one. The isosurface that forms the boundary surface of an object is usually defined as a set of connected triangle patches. The technique for generating an isosurface from volumetric data is a useful tool in many areas.

Many methods for generating an isosurface have been reported. Most of these construct an isosurface as an approximated polyhedron composed of small triangle patches. The marching cubes (MC) method [1] is a well known fundamental method for isosurface construction from volumetric data sampled with an orthogonal cubic lattice. The MC method generates triangle patches in a cube composed of eight adjacent lattice points (Fig. 1). The vertices of each patch are located on the edge of the cubic lattice. Many methods improving upon the MC method have been developed [2]-[10]. Some of the methods deal with volumetric data sampled with other types of lattice to improve the quality of the resulting surfaces. Most of these methods use a body-centered cubic (BCC) lattice or a face-centered cubic (FCC) lattice [11]-[16]. The quality of an isosurface depends on the triangle patches – patches with very acute angles or that are too small lower the surface quality. The aspect ratio of a triangle is thus used as a criterion to evaluate the shape of each patch, and is often used to evaluate the isosurface quality [11][14][17]-[19]. A regular triangle has a maximum aspect ratio of 1. As a triangle becomes more acute, the aspect ratio approaches 0. When a triangle patch is very small, though, even if it has a good aspect ratio it does not help improve the isosurface quality. Therefore, two requirements must be met to realize a fine isosurface.

- The isosurface must contain many triangle patches with good aspect ratios.
- The surface must contain many triangle patches of almost the same size.

We propose a new method for generating an isosurface from volumetric data sampled with an FCC lattice. The MC method uses a cube, which is widely used with common volumetric data (Fig. 1). In contrast, our proposed method uses two types of polyhedron – two tetrahedrons and an octahedron. Such a polyhedron is called the cell. A tetrahedral cell is composed of four adjacent lattice points and an octahedral cell is composed of six adjacent lattice points (Fig. 2). Most methods generate each triangle patch within a polyhedral cell. Our method generates it among adjacent cells. It turns out that our method leads to a high quality isosurface in terms of the above requirements. We experimentally constructed surfaces to test the effectiveness of our method. In these experiments, we algebraically generated two kinds of volumetric data and compared the effectiveness of the MC method and that of a method using an FCC lattice which we previously proposed [12][13] with the effectiveness of our new method.

2. RELATED WORK

2.1. Marching Cubes method

The MC method is used for isosurface construction from volumetric data sampled with an orthogonal cubic lattice [1]. The MC method constructs an isosurface as an approximate

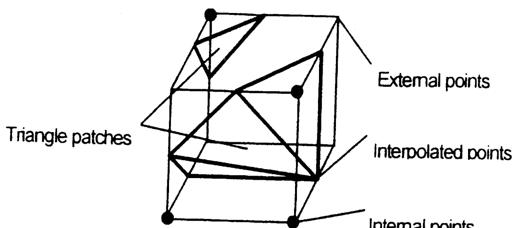

Figure 1. Triangle patch generation in a cubic cell with the MC method

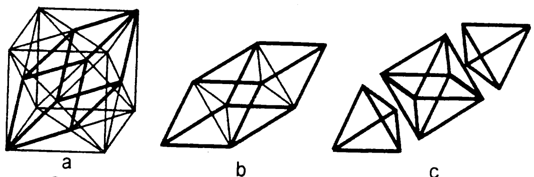

Figure 2. Parallelepiped tessellation: (a) and (b) show a parallelepiped cell in an FCC lattice, and (c) shows two types of polyhedral cell that are used in a method with an FCC lattice.

polyhedron composed of small triangle patches. Each of the triangle patches is generated in a cubic cell as shown in Fig. 1. The cubic cell is composed of eight adjacent lattice points. When one of two lattice points on an edge of the cube is an internal point of an object domain and the other is external, such a pair of lattice points is called a boundary pair. The MC method defines each vertex of the patch between the boundary pair by means of linear interpolation. 256 (i.e., $= 2^8$) configurations of a cubic cell can be considered for different arrangements of the internal and external lattice points. Accordingly, a pre-defined table containing the correspondence between the configuration and the way of triangulation can be used to reduce the computational cost. However, defining such a large table incurs a large cost. These 256 configurations are integrated into 14 fundamental configurations by taking into account symmetry, rotation, and inversion. There are, however, some ambiguous triangulations among the configurations. Therefore, when each of two adjacent cubic cells belongs to particular configurations, the MC method generates a topological hole in the resulting surface. Many methods have been developed to prevent this ambiguity [4]-[10]. One of the simplest solutions is exceptional definition of the triangulations for each of the particular configurations [6][7][9]. Tetrahedral tessellation methods [20]-[22] are also a solution. They divide the cubic cell into tetrahedral cells. The tetrahedral cell has only 16 possible triangulations, and these are reduced to three through symmetry.

2.2. Isosurface Construction with Other Lattices

Methods not based upon the orthogonal cubic lattice have also been developed. These methods deal with a face-centered cubic (FCC) lattice or a body-centered cubic (BCC) lattice. To generate triangle patches, these methods use polyhedral cells of various shapes. In the methods dealing with a BCC lattice,

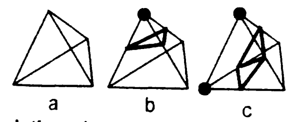

Figure 3. Triangulations in a tetrahedral cell

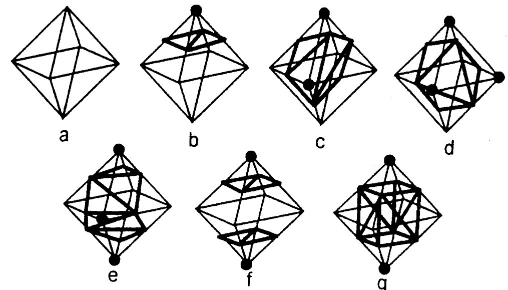

Figure 4. Triangulation in an octahedral cell: there are two different triangulations (f) and (g) for the same configuration.

tetrahedral tessellation [14][15], octahedral tessellation, and hexahedral tessellation [16] have been reported.

The FCC lattice structure is the closest packing structure in the sphere-packing problem [23]. There have been reports that the topological understanding of a figure on an FCC lattice is simpler than that for one on an orthogonal cubic lattice [24][25]. We earlier proposed a method for the FCC lattice [12][13]. This method does not create false holes, and improves the geometrical property of the resulting patches [11]. It uses both a regular octahedral cell and a regular tetrahedral cell. These cells are divided from a parallelepiped cell composed of eight adjacent lattice points (Fig. 2). The triangulations in each of these cells are achieved in the same manner as with the MC method. There are three possible triangulations in the tetrahedral cell (Fig. 3), and the octahedral cell has 64 possible triangulations, which are reduced to seven through symmetry (Fig. 4). Ambiguous triangulations are shown in Figs. 4(f) and (g). In this octahedral cell, the topology of the resulting surface may not be consistent. This method, however, does not generate topological holes if either cell is used for triangulation. Furthermore, continuity of the resulting surface is entirely retained. Therefore, the particular triangulation shown in Fig. 4(f) is used in this paper.

3. ALGORITHM

3.1. Isosurface Construction

Our method uses the same polyhedral cells as were used in our previous method for an FCC lattice [12][13]. The patch generation, however, differs from that in most other methods. Our proposed method generates each triangle patch among adjacent polyhedral cells, while most methods generate the patch within the polyhedral cell. Each boundary pair is shared among four polyhedral cells (i.e., two octahedral cells and two tetrahedral cells) as shown in Fig. 5. Our method generates two triangle patches between the boundary pair. The vertex of each pair of patches lies within either two octahedral cells or two tetrahedral cells. These triangle patches tend to have a good

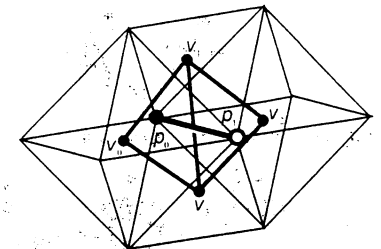

Figure 5. Triangle patch generation with our method: Two triangle patches $v_0v_1v_3$ and $v_1v_2v_3$ are generated among four adjacent polyhedral cells that have the same boundary pair p_0p_1.

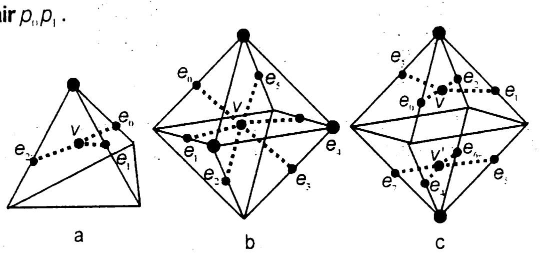

Figure 6. Definition of the vertex in polyhedral cells with our method: A vertex v is generated at the barycenter of points e_i that are interpolated on the edges between the boundary pairs through our previous FCC method.

aspect ratio because of the relative position of these three polyhedrons. Each vertex is defined as the barycenter of the points calculated by our previous method for an FCC lattice on the polyhedral cell. This reduces the number of very thin or small triangle patches. The algorithm can be described as follows.

1. When an octahedral cell has more than one boundary pair, detect the tetrahedral cells that share the boundary pair.
2. For each detected tetrahedral cell and the octahedral cell, temporarily calculate the edge points on the cell by our previous method.
3. A vertex in each of the cells is located at the barycenter of these temporary points on the cell as shown in Figs. 6(a) and (b). Two vertices are generated inside of the octahedral cell from the two sets of the temporary points only in the case shown in Fig. 6(c). These vertices correspond to the two separate polygons shown in Fig. 4(f).
4. Triangle patches are generated from these vertices so that every patch has a vertex in the octahedral cell and two vertices in the tetrahedral cells (Fig. 7). In the case of Fig. 7(f), two polygons are generated.

3.2. Implementation

To reduce the computational cost, our method can also use a predefined table containing the correspondence between the octahedral cell configurations and the ways of triangulation. In our method, there are six fundamental triangulations for the octahedral cells shown in Fig. 7. These correspond to the triangulations of our previous method with an FCC lattice as shown in Figs. 4(a) to (f). There are only three polygon shapes (i.e., tetragon, hexagon, and octagon) that have a vertex in the

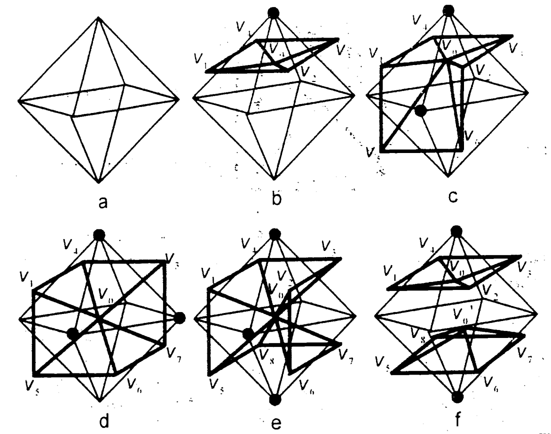

Figure 7. Triangulations around an octahedral cell: The vertices located in an octahedral cell are represented by v_0 and v_0'. Other vertices located in each tetrahedral cell adjoining the octahedral cell are represented by $v_i (i \neq 0)$ which correspond to v_i shown in Fig. 8.

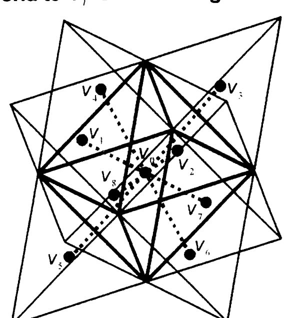

Figure 8. An octahedral cell and eight adjacent tetrahedral cells: The vertex located in the octahedral cell is represented by v_0. Other vertices located in each tetrahedral cell are represented by $v_i (i \neq 0)$ which correspond to v_i shown in Fig. 7.

octahedral cell. Moreover, each polygon can be triangulated uniquely, whereas several possible triangulations exist in other methods.

Each of the vertices in the tetrahedral cells may have to be calculated three or four times because each tetrahedral cell adjoins four octahedral cells; however, the computing cost can be reduced by storing in memory the status of each vertex inside of the tetrahedral cell when it is referred to for the first time.

4. RESULTS AND DISCUSSION

4.1. Comparison Conditions

To compare our proposed method with existing methods (i.e., the MC method [1] and our previous FCC method [12][13]) in terms of the geometrical features of the resulting surfaces, we used two kinds of volumetric data (i.e., a spherical surface and a complicated one generated by meta-ball [26]). The volumetric data were sampled for three different resolutions. In each

Table 1. Quantitative results for the spherical surface

Resolution	Method	Internal points	Patches	Mean patch area	Standard deviation
Low	MC	305	680	0.3260	0.2304
	Previous FCC	321	1508	0.1477	0.1996
	Our method	321	1512	0.1462	0.0564
Middle	MC	2517	2648	0.0819	0.0515
	Previous FCC	2587	6260	0.0359	0.0577
	Our method	2587	6264	0.0358	0.0178
High	MC	20341	10616	0.0212	0.0124
	Previous FCC	20473	24644	0.0092	0.0143
	Our method	20473	24648	0.0091	0.0043

Table 2. Quantitative results for the complicated surface

Resolution	Method	Internal points	Patches	Mean patch area	Standard deviation
Low	MC	9929	12224	0.3325	0.2004
	Previous FCC	9963	27236	0.1529	0.2285
	Our method	9963	27244	0.1452	0.0677
Middle	MC	79586	50164	0.0833	0.0499
	Previous FCC	79571	112752	0.0374	0.0560
	Our method	79571	112728	0.0368	0.0161
High	MC	636672	201392	0.0209	0.0125
	Previous FCC	636567	454808	0.0093	0.0139
	Our method	636567	454780	0.0092	0.0040

experiment, for the same volume data, the resulting surfaces were compared in terms of five criteria: (1) the number of internal points, (2) the number of triangle patches, (3) the surface area of each triangle patch, (4) the aspect ratio of each triangle patch, (5) the appearance of the resulting surface.

The surface area of a triangle patch was calculated under the condition that the distance between two adjacent lattice points on the orthogonal cubic lattice could become 1. The aspect ratio is one of the criteria used to evaluate the shape of a triangle. A triangle that is similar to a regular triangle has a good aspect ratio of close to 1, while a thin triangle has a poor aspect ratio of close to 0. There are many definitions of the aspect ratio. In this paper, we define the aspect ratio A as $A = 2r/R$, where r and R represent, respectively, the radius of the inscribed circle and of the circumscribed circle. Triangle patches with a poor aspect ratio and very small triangle patches degrade the surface appearance, thus these criteria are important ones.

To ensure a fair comparison, we considered the distance between neighboring lattice points so that the volume of the rhombic dodecahedron that is the Voronoi polyhedron for the FCC lattice point and the volume of the cube that is the Voronoi polyhedron for the orthogonal cubic lattice point would be the same. Thus, the sampling point densities on the two types of lattice were almost the same.

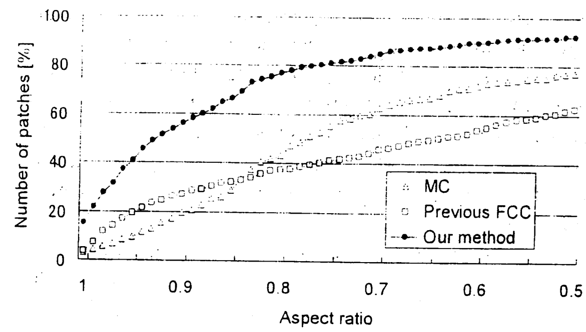

Figure 9. Cumulative histogram of the aspect ratio for the spherical surface to the highest resolution shown in Table 1

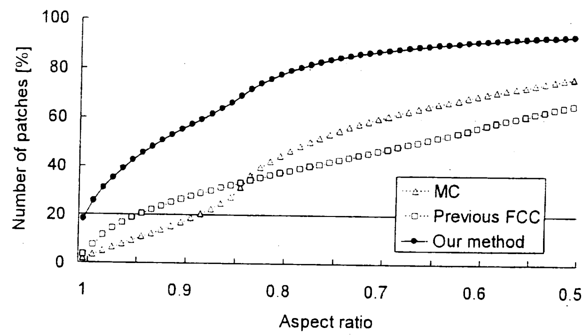

Figure 10. Cumulative histogram of the aspect ratio for the complicated surface to the highest resolution shown in Table 2

4.2. Resulting Surfaces

Figures 11 and 12 show the experimental results from the isosurface construction in the case of the lowest resolution. Figures 9 and 10 are cumulative histograms of the aspect ratio in the case of the highest resolution, and Tables 1 and 2 show the quantitative results for each resolution. The surface obtained through our method seems to be the finest in appearance from among the three methods. Some jags can be seen in the surface obtained through the MC method and our previous FCC method; these were due to the existence of very thin or small triangle patches. In contrast, few jags appeared in the resulting surface when our new method was used. Two measurements confirmed this finding. First, the standard deviation of the patch area was smaller than that with our previous FCC method (Tables 1 and 2). Second, our method generated a much larger number of almost regular triangle patches (Figs. 9 and 10).

The three methods were almost equivalent in the number of internal points for each resolution (Tables 1 and 2). In each case, our method and the previous FCC method generated almost the same number of patches. However the standard deviation against the mean surface area of our method was smaller than that of the previous FCC method. This demonstrates that our method generated fewer triangle patches that were too small or too big. Figures 9 and 10 show that the resulting surface when our method was used had a lot of good aspect ratio triangle patches. When our new method was used, the percentage of patch number with a good aspect ratio (i.e., > 0.9) was the highest and the percentage of patch number with a poor aspect ratio (i.e., < 0.7) was the lowest among these three methods. This histogram feature was not depend on the surface geometry and could be seen for all other resolutions. Thus, for the reasons given above, we could conclude that our proposed method generates many good aspect ratio triangle patches of almost uniform size.

Our proposed method, however, generates more than double the number of triangle patches generated by the MC method (Tables 1 and 2). Therefore, a means of triangle patch reduction that removes poor aspect ratio triangle patches may be needed.

5. CONCLUSION

In this paper, we have proposed a new method for constructing high quality isosurfaces from volumetric data sampled with an FCC lattice, and have demonstrated the effectiveness of our method in experiments by comparing the resulting surfaces with those from existing methods in terms of geometrical criteria. To summarize, our method provides the following benefits.

- The quality of the resulting surface is greatly improved through the use of many good aspect ratio triangle patches that tend to be uniformly large.
- The resulting surface is composed of only three types of polygon that are uniquely triangulated.
- The resulting surface does not have topological holes as are generated by the MC method, and the topology is identical to that obtained with our previous method using an FCC lattice.
- A computational cost for isosurface construction comparable to that with existing methods can be obtained by using a pre-defined table regarding the triangulation.

Our future work will include isosurface construction from computed tomography data using our method, and development of a method for removing any triangle patches with a poor aspect ratio that are generated by our method.

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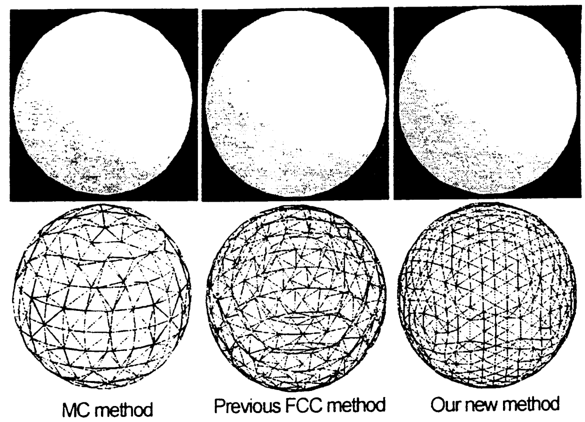

Figure 11. Resulting surfaces of each method for the spherical surface: these surfaces correspond to the lowest resolution shown in Table 1.

REFERENCES

[1] W. E. Lorensen and H. E. Cline. Marching cubes: A high resolution 3d surface construction algorithm. *Computer Graphics*, 21(4): 163-169, Jul. 1987.

[2] G. Wyvill, C. McPheeters, and B. Wyvill. Data structure for soft objects. *The Visual Computer*, 2(4): 227-234, Feb. 1986.

[3] A. A. Pasko, V. V. Pilyugin, and V. N. Pokrovskiy. Geometric modeling in the analysis of trivariate functions. *Computers and Graphics*, 12(3/4):457–465, 1988.

[4] P. Ning and J. Bloomenthal. An evaluation of implicit surface tilers. *IEEE CG and Applications*, 13(6): 33-41, Nov. 1993.

[5] A. Gelder and J. Wilhelms. Topological considerations in isosurface generation. *ACM Trans. On Graphics*, 13(4): 337-375, Oct. 1994.

[6] C. Montani, R. Scateni, and R. Scopigno. A modified look-up table for implicit disambiguation of Marching Cubes. *The Visual Computer*, 10:353-355, 1994.

[7] T. Takahashi and T. Yonekura. Surface construction method of digital object considering four types of connectivity. *IEICE Trans.*, J82-D-II(11): 2160-2164, Nov. 1999.

[8] T. Ishii, T. Yasuda, S. Yokoi, and J. Toriwaki. An improved algorithm of the marching cubes method. *IEICE Technical Report*, PRU92-86: 63-70, Dec. 1992.

[9] T. Nagae, T. Agui, and H. Nagahashi. Object surface construction from volume data with appropriate topology. *IEICE Trans.*, J76-D-II(8): 1704-1711, Aug. 1993.

[10] T. Nagae, T. Agui, and H. Nagahashi. Orientable closed surface construction from volume data. *IEICE Trans.*, E76-D(2): 269-273, Feb. 1993.

[11] T. Takahashi and T. Yonekura. Geometrical properties of isosurface from data set on face-centered-cubic lattice. *IEICE Trans.*, J86-A(7): 806-811, 2003.

[12] T. Takahashi, T. Yonekura, and Y. Kishi. A method of equi-valued surface construction from volume data on face centered cubic lattice. *IEICE Trans.*, J85-A(11): 1313-1323, Nov. 2002.

[13] T. Takahashi and T. Yonekura. Isosurface construction from a data set sampled on a face-centered-cubic lattice. *Proc. of ICCVG2002*, 2: 754-763, Sept. 2002.

[14] G. M. Treece, R. W. Prager, and A. H. Gee. Regularised marching tetrahedra: improved isosurface extraction. *Computers and Graphics*, 23(4): 583-598, 1999.

[15] S. L. Chan and E. O. Purisima. A new tetrahedral tesselation scheme for isosurface. *Computers and Graphics*. 22(1): 83-90. 1998.
[16] H. Carr. T. Theußl. and T. Möller. Isosurface on optimal regular samples. *IEEE TCVG VisSym 2003*. 39-48. May 2003.
[17] Z. Wood. M. Desbrun. P. Schröder and D. E. Breen. Semi-regular mesh extraction from volumes. *Proc. of IEEE VIS2000*, 275-282. Oct. 2000.
[18] I. Guskov. A. Khodakovsky. P. Schröder. and W. Sweldens. Hybrid meshes: Multiresolution using regular and irregular refinement. *Proc. of ACM SCG2000*, 264-272. June 2002.
[19] P. S. Heckbert and M. Garland. Optimal triangulation and quadric-based surface simplification. *Computational Geometry*. 14(1-3):49-65. 1999.
[20] B. A. Payne and A. W. Toga. Surface mapping brain function on 3d models. *IEEE CG and Application*, 5(10): 33-41. Sep. 1990.
[21] A. Doi and A. Koide. An efficient method of triangulating equi-valued surfaces by using tetrahedral cells. *IEICE Trans.*. E74(1): 214-224. Jan. 1991.
[22] A. Doi. S. Suzuki. K. Oyamada. and S. Sannakanishi. Error evaluation and visualization of tetrahedral subdivision approach for trilinear interpolation. *Journal of IIEEJ*. 27(4):298-305. 1998.
[23] J. H. Conway and N. J. A. Sloane. Sphere packings. lattices. and groups. 2nd ed. *New York: Springer-Verlag*. 1993.
[24] T. Yonekura. S. Yokoi. J. Toriwaki. and A. Fukumura. Topological properties of three dimensional digital image sampled on the face-centered cubic lattice. *IEICE Trans.*, J65-D(7): 920-927. Jul. 1982.
[25] T. Yonekura. S. Yokoi. J. Toriwaki. and A. Fukumura. Connectivity and euler number of figures in the digital three-dimensional space. *IEICE Trans.*, J65-D(1): 22-29. Jan. 1982.
[26] H. Nishimura. M. Hirai. T. Kawai. T. Kawada. I. Shirakawa. and K. Omura. Object modeling by distribution function and a method of image generation. *IEICE Trans.*, J68-D(4):718-725. Apr. 1985.

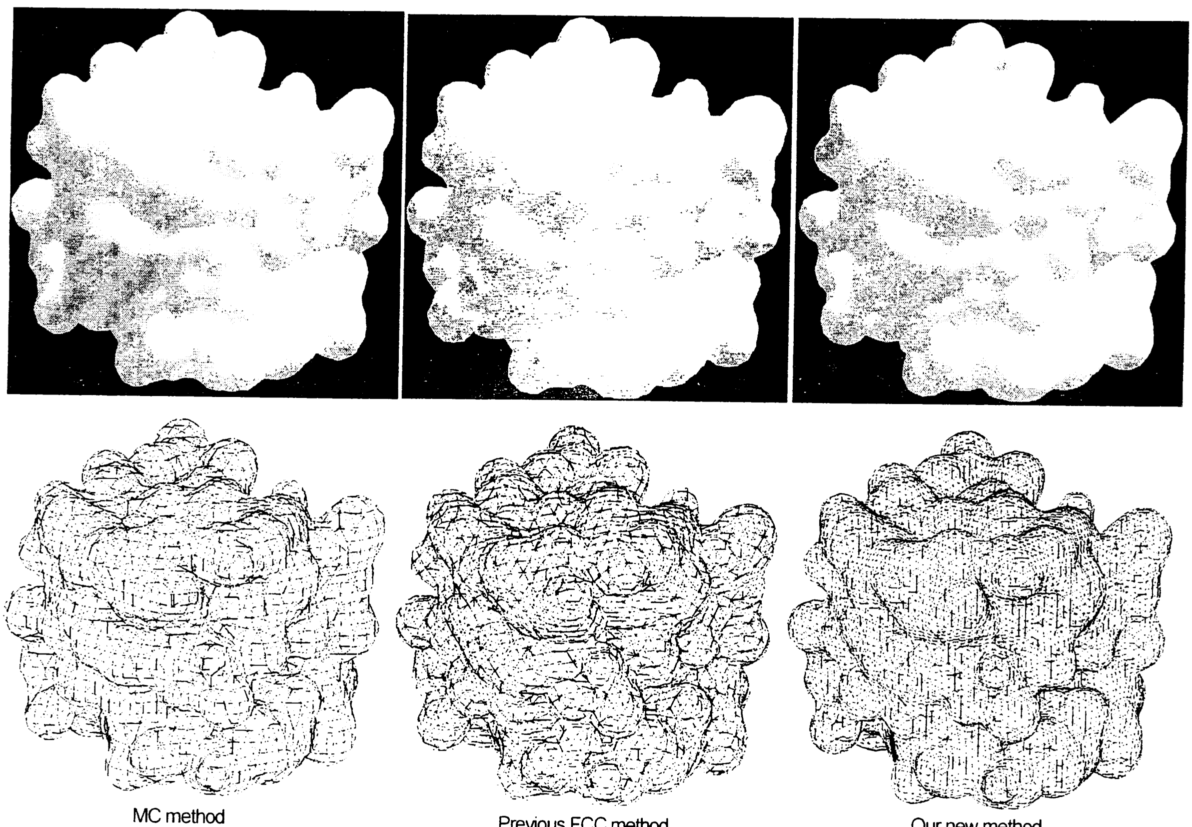

Figure 12. Resulting surfaces of each method for the complicated surface: these surfaces correspond to the lowest resolution shown in Table 2.